09

2014학년도 9월 평가원(B형) 21번

자연수 n에 대하여 함수 $y=f(x)$를 매개변수 t로 나타내면

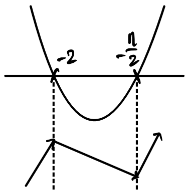
$$\begin{cases} x = e^t \xrightarrow{\quad '\quad} e^t \\ y = (2t^2+nt+n)e^t \xrightarrow{\ '\ } \{2t^2+(n+4)t+2n\}e^t \end{cases} \longrightarrow \frac{dy}{dx} = (t+2)(2t+n)$$

이고, $x \geq e^{-\frac{n}{2}}$일 때 함수 $y=f(x)$는 $x=a_n$에서 최솟값 b_n을 갖는다.
$\dfrac{b_3}{a_3}+\dfrac{b_4}{a_4}+\dfrac{b_5}{a_5}+\dfrac{b_6}{a_6}$의 값은? [4점]

① $\dfrac{23}{2}$ ② 12 ③ $\dfrac{25}{2}$

④ 13 ⑤ $\dfrac{27}{2}$

i) $-\frac{n}{2} > -2$: $n<4$



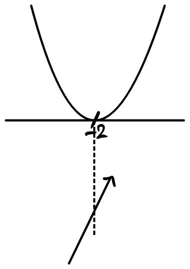
$t=-\frac{n}{2} \rightarrow e^{-\frac{n}{2}}$ 에서 극소 $\longrightarrow x=e^{-\frac{3}{2}}$ 에서 최소

$\longrightarrow a_3 = e^{-\frac{3}{2}}$, $b_3 = \left\{2\cdot\left(-\frac{3}{2}\right)^2 + 3\cdot\left(-\frac{3}{2}\right)+3\right\}e^{-\frac{3}{2}}$

$= 3e^{-\frac{3}{2}}$

$\therefore \dfrac{b_3}{a_3} = 3$

ii) $-\frac{n}{2}=-2$: $n=4$



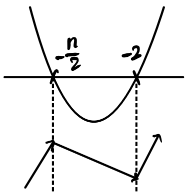
$t=-2 \longrightarrow x=e^{-2}$ 에서 최소

$\longrightarrow a_4=e^{-2}$, $b_4=\{2\cdot(-2)^2+4\cdot(-2)+4\}e^{-2}$

$=4e^{-2}$

$\therefore \frac{b_4}{a_4}=4$

iii) $-\frac{n}{2}<-2$: $n>4$



$t=-\frac{n}{2} \longrightarrow e^{-\frac{n}{2}}$ 에서 극소 $\longrightarrow x=e^{-2}$ 에서 최소

$\longrightarrow a_n=e^{-2}$, $b_n=\{2\cdot(-2)^2+n\cdot(-2)+n\}e^{-2}$

$=(8-n)e^{-2}$

$\therefore \frac{b_n}{a_n}=8-n$

$\therefore \frac{b_3}{a_3}+\frac{b_4}{a_4}+\frac{b_5}{a_5}+\frac{b_6}{a_6}=12$